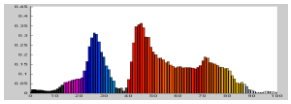


Dr. Qadri Hamarsheh

Signals and Systems

Lecture 7

Time Domain Models of Systems

Outline

- **Input –Output representation of D-T systems examples.**
- **General class of systems.**

Input –output representation of D-T systems examples:

a- $N-$point MA average filter:

MA average filter can be expressed by the following equation:

$$y[n]=\frac{1}{N}\left[x[n]+x[n-1]+...+x[n-N+1]\right]=\frac{1}{N}\sum_{k=o}^{N-1}x[n-k],$$

where N is a positive integer, $y[n]$ is the output signal in time domain and $x[n]$ is the input signal in time domain.

Using this equation, we can **generalize** it to represent a large class of causal LTI system, as shown in the following modified equation:

$$y[n]=w_0x[n]+w_1x[n-1]+w_2x[n-2]...+w_{N-1}x[n-N+1]=\sum_{k=o}^{N-1}w_k\,x[n-k].$$

In this case, we use for each sample some w_k that called the weight (the percentage of the participation for the given sample in given n sample output).

$w_k \Rightarrow (w_1,w_2,...,w_{N-1})$ are real numbers of the linear combination

If all weights are equal to $\frac{1}{N}$, we get equation for MA Filter

b- Exponentially Weighted Moving Average (EWMA) Filter:

Using the general form, the $N-$point **EWMA** filter defined by:

$$y[n]=\sum_{k=o}^{N-1}a\left(b^k\,x[n-k]\right)$$

Where

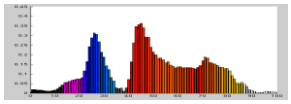
b is a real number within the range : $0<b<1$.

a is a positive number that equals: $a=\frac{1-b}{1-b^N}$

If $b=0$, then $a=1$ and the equation reduced to $y[n]=x[n]$
This means that no filtering process of the input signal

According to the previous section, the weights of EWMA filter will be defined as the following

$w_k=ab^k$, $k=0,1,2,...,N-1$. (Note that these weights are exponentially based weights and they are decrease in values as k increase in value).



If $N=5;\quad b=0.7$ then $a=\dfrac{1-0.7}{1-0.7^5}=\dfrac{0.3}{0.832}=0.3606$

$$w_0=a=0.3606$$

$$w_1=ab=0.2524$$

And using $w_k=ab^k \Rightarrow \quad w_2=ab^2=0.1767 \quad \Rightarrow$

$$w^3=ab^3=0.1237$$

$$w_4=ab^4=0.0866$$

$$y[n]=0.3606x[n]+0.2524x[n-1]+0.1767x[n-2]+0.1237x[n-3]+0.0866x[n-4].$$

In EWMA filter, a larger weight is given to the more recent samples of the input during the computation of the output $y[n]$, and in MA filter, all the samples have the same weight.

$$y[n]=0.2x[n]+0.2x[n-1]+0.2x[n-2]+0.2x[n-3]+0.2x[n-4]$$

General class of systems

General class of system is given by replacement the upper index $N-1$ in the summation with n as follows

$$y[n]=\sum_{k=o}^{n} w_k\, x[n-k], \quad n\geq 0$$

Using this equation we can express any causal LTI D-T system with input $x[n]=0$, for all $n<0$

Matlab example:

```
%MAandEWMA.m script program.
%plot the values of N-point MA Filter and
%the values of N-point EWMA Filter
%Use 'input' matlab command to control the input of the length
%of the two filters (N), the parameter B.
% B must be 0 < B < 1, this condition must be validate

% Get the input from the user
N = input('Please, enter the length for MA and EWMA Filters');
B = input ('Please, enter the value for B, 0 < B < 1');
if (B <= 0 | B>=1)
    error('B must be greater than 0 and less than 1');
end
% MA Filter coefficients generation
n = 0:N-1;
MAW = (1/N)*ones(size(n));
subplot(2,1,1);
stem(n,MAW,'filled');
title('Impulse Response of MA filter');
xlabel('samples');
ylabel('MA Coefficients');
axis auto;
% EWMA Filter coefficients generation
A = (1-B)/(1-B^N);
```

```
EWMAW = A*B.^n; %EWMA Coefficients
subplot(2,1,2);
stem(n, EWMAW, 'filled');
title('Impulse Response of EWMA Filter ');
xlabel ('samples');
ylabel ('EWMA Coefficients');
axis auto
```

The run of **MAandEWMA.m** script program will be the following:

```
Please, enter the length for MA and EWMA Filters 15
Please, enter the value for B, 0 < B < 1 0.7
```

If the user enters 15 for N and 0.7 for b, then the output plot will be as shown in figure 4-1.

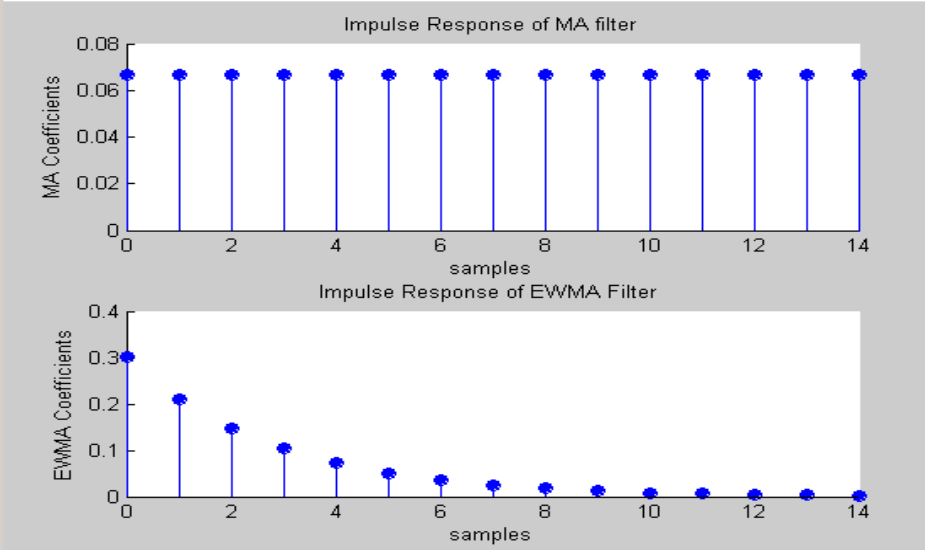


Figure 4-1